Differential Equations

LAB 3

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**Question 1**

Based on calculations done by hand we deduce these equations for the function

function [Ac, As, omegas, Af] = secondUndampedSolver(B, A, omegaf, y0, v0)

Af = A/(B-(omegaf^2));

Ac = y0 - Af;

omegas = sqrt(B);

As = v0/omegas;

end

example

[Ac,As,omegas,Af] = secondUndampedSolver(4,0,0,1,0)

Ac =

1

As =

0

omegas =

2

Af =

0

**Question 2**

w=4

[Ac,As,omegas,Af] = secondUndampedSolver(9,1,4,0,0)

Ac =

0.1429

As =

0

omegas =

3

Af =

-0.1429

w=3.1

[Ac,As,omegas,Af] = secondUndampedSolver(9,1,3.1,0,0)

Ac =

1.6393

As =

0

omegas =

3

Af =

-1.6393

w=3.01

[Ac,As,omegas,Af] = secondUndampedSolver(9,1,3.01,0,0)

Ac =

16.6389

As =

0

omegas =

3

Af =

-16.6389

w=3

[Ac,As,omegas,Af] = secondUndampedSolver(9,1,3,0,0)

Ac =

-Inf

As =

0

omegas =

3

Af =

Inf

Comment

As w decreases in value the values of Ac and Af increases. In w=3 since we’re dividing by zero in Af we got the value of Af as infinity while Ac -infinity. So w=3 went wrong

**Question 3**

The method didn’t work as the guess being used doesn’t work, if we used the guess for y particular as y=Acos(3t) or y=Acos(3t) + Bsin(3t) by doing the equation by hand you get a wrong statement which is 0A=1 so the statement is wrong therefore following this method with these guesses don’t work for this omega. As cos(3t) is actually one of the solutions.

So a guess of y=Atcos(3t) + Btsin(3t) is being used instead to solve this equation. The solution at the end after solving by hand is y= tsin(3t)/6 for w=3

**Question 4**

syms y(t)

[V] = odeToVectorField(diff(y, 2) == (cos(3\*t) - (9\*y)));

M = matlabFunction(V,'vars', {'t','Y'});

sol = ode45(M,[0,20],[0 0]);

figure(1)

fplot(@(x)deval(sol,x,1), [0, 20]);

hold off

figure(2)

fplot(@(x) (x\*sin(3\*x))/6 ,[0, 20],'r');

Figure1



Figure2



The two figures from the solution obtained in matlab(figure1) and the solution obtained by solving(figure2) are the exact same and they match so this is the solution of w=3

**Question 5**

syms y(t)

Dy = diff(y);

ode = diff(y,t,2) == (cos(3\*t) - (9\*y));

cond1 = y(0) == 0;

cond2 = Dy(0) == 0;

conds = [cond1 cond2];

ySol(t) = dsolve(ode,conds);

ySol = simplify(ySol)

**ySol(t) =**

**(t\*sin(3\*t))/6**

The solution obtained through this method matches the solution obtained before therefore this method is valid in this case.

**Question 6**

ω = 2300

syms y(t)

Dy = diff(y);

ode = diff(y,t,2) == (((2400-2300)\*2400\*cos(2300\*t)) - ((2400^2)\*y));

cond1 = y(0) == 0;

cond2 = Dy(0) == 0;

conds = [cond1 cond2];

ySol(t) = dsolve(ode,conds);

ySol = simplify(ySol)

M = matlabFunction(ySol,'vars', {'t'});

fplot(M ,[0, 2])

ySol(t) =

(24\*cos(2300\*t))/47 - (24\*cos(2400\*t))/47

Chart, bar chart

Description automatically generated

ω = 2350

syms y(t)

Dy = diff(y);

ode = diff(y,t,2) == (((2400-2350)\*2400\*cos(2350\*t)) - ((2400^2)\*y));

cond1 = y(0) == 0;

cond2 = Dy(0) == 0;

conds = [cond1 cond2];

ySol(t) = dsolve(ode,conds);

ySol = simplify(ySol)

M = matlabFunction(ySol,'vars', {'t'});

fplot(M ,[0, 2])

ySol(t) =

(48\*cos(2350\*t))/95 - (48\*cos(2400\*t))/95

Chart, bar chart

Description automatically generated

ω = 2390

syms y(t)

Dy = diff(y);

ode = diff(y,t,2) == (((2400-2390)\*2400\*cos(2390\*t)) - ((2400^2)\*y));

cond1 = y(0) == 0;

cond2 = Dy(0) == 0;

conds = [cond1 cond2];

ySol(t) = dsolve(ode,conds);

ySol = simplify(ySol)

M = matlabFunction(ySol,'vars', {'t'});

fplot(M ,[0, 2])

ySol(t) =

cos(2400\*t)\*(cos(10\*t)/2 + cos(4790\*t)/958) - (240\*cos(2400\*t))/479 + sin(2400\*t)\*(sin(10\*t)/2 + sin(4790\*t)/958)

A picture containing chart

Description automatically generated

The three figures proof that as the value of omega increases then the solution’s frequency decreases. As the last figure has waves with wider forms as the frequency is less unlike the first figure where the frequency was high and the waves formed had shorter wavelength.

**Question 7**

Part a

syms y(t)

Dy = diff(y);

ode = diff(y,t,2) == -(16\*y)-(Dy);

cond1 = y(0) == 10;

cond2 = Dy(0) == 0;

conds = [cond1 cond2];

ySol(t) = dsolve(ode,conds);

ySol = simplify(ySol)

M = matlabFunction(ySol,'vars', {'t'});

fplot(M ,[0, 2])

ySol(t) =

(10\*exp(-t/2)\*(21\*cos((3\*7^(1/2)\*t)/2) + 7^(1/2)\*sin((3\*7^(1/2)\*t)/2)))/21



Part b

syms y(t)

Dy = diff(y);

ode = diff(y,t,2) == -(16\*y)-(10\*Dy);

cond1 = y(0) == 10;

cond2 = Dy(0) == 0;

conds = [cond1 cond2];

ySol(t) = dsolve(ode,conds);

ySol = simplify(ySol)

M = matlabFunction(ySol,'vars', {'t'});

fplot(M ,[0, 2])

ySol(t) =

(10\*exp(-8\*t)\*(4\*exp(6\*t) - 1))/3



The first equation represents the pendulum in water as the graph represents a wave of a moving pendulum as water has less viscosity than molasses so the pendulum moves back and forth. The second equation represents the pendulum in molasses as it has a higher viscosity and the pendulum cannot move freely so it slows down until it stops